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A Relation Between Steam Quality and Void Fraction in Two-Phase Flow

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The prediction of void fraction in two-phase flow is very important to determine reactivities of boiling water reactors and circulation rates of water in boilers.

Most of the work to date has been of an experimental nature. Theoretical attempts for this problem have been made by Levy (*1*) and Bankoff (*2*). The results of these treatments agree with the results of experiments under some flow conditions, but they are not adequate.

A theoretical treatment which deals with the annular flow regime is proposed in this paper. To deal with a two-phase flow in which water evaporates into vapor the two-phase flow system is considered to consist of three regions. One of the regions is occupied by liquid only, another is occupied by vapor, and a third region provides for the phase exchanges (evaporation). From the equations of force balance for these three regions, and an assumption for the forces between these regions, a relationship between steam quality and void fraction and a relationship between steam quality and two-phase frictional losses are derived.

The effects of entrained liquid quantity on the void fractions as well as on the momentum changes are neglected because in accordance with Dukler (*3*) the entrained liquid quantity is small compared with the quantity of the film along the wall, unless the gas velocity in the core exceeds 100 ft./sec.

The derived formulas offer explanations for differences arising between the use of vertical and horizontal pipes, the effect of the total flow rate for vertical flow, and the effect of evaporation on the relationship between the steam quality and void fraction.

BASIC EQUATIONS

An evaporating two-phase flow is considered to consist of three regions, *A*, *B*, and *C* (see Figure 1). Regions *A* and *C* are respectively occupied by liquid and vapor whose velocity can change between sections *I* and *II*, but the mass flow rate does not change. Region *B* is occupied by liquid only at section I, and all the liquid vaporizes into vapor between section I and II.

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The equation of the force balance for each region has been simplified as follows:

$$\frac{1}{g} G_l \frac{dV_l}{dL} = A_l \frac{dP}{dL} - A_l \rho_l \sin\theta - \frac{1}{g} A\, F_{wl} + \frac{1}{g} F_{lt} \tag{1}$$

$$\frac{1}{g} V_R \frac{dG_g}{dL} = \frac{1}{g} (F_{gt} - F_{lt}) \tag{2}$$

$$\frac{1}{g} G_g \frac{dV_g}{dL} = A_g \frac{dP}{dL} - A_g \rho_g \sin\theta - \frac{1}{g} F_{gt} \tag{3}$$

Adding Equations (1), (2), and (3) one obtains the equation of force balance for the whole two-phase flow system:

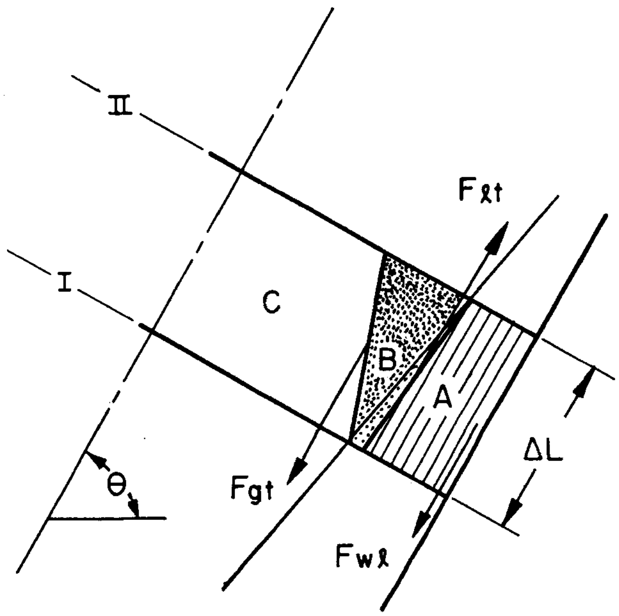

Fig. 1. Model of evaporating two-phase flow.

$$\frac{1}{g}\frac{d}{dL}(G_lV_l+G_gV_g)=A\frac{dP}{dL}-(A_l\rho_l+A_g\rho_g)\sin\theta-\frac{1}{g}AF_{wl}$$

When there is no vaporization along the channel (air-water system can be the case), the force between the phases is assumed to be proportional to the square of the relative mean velocity between the gas and the liquid, and to the interfacial area of the gas and the liquid. It is suggested by the Prandtl mixing length theory that the force is also proportional to the square of the distance between the pipe wall and liquid-gas interface. Thus F_{gt} and F_{lt} are assumed as follows when $dG/dL = 0$:

$$F_{gt_o}=F_{lt_o}=C_1\sqrt{\alpha}(1-\sqrt{\alpha})^2\,V_R^2 \qquad (4)$$

where C_1 can be a function of the flow condition. When $dG/dL \neq 0$, F_{gt} and F_{lt} are assumed to be

$$F_{gt}=C\,V_R^2+S\,V_R\frac{dG_g}{dL} \qquad (5)$$

$$F_{lt}=C\,V_R^2-(1-S)V_R\frac{dG_g}{dL} \qquad (6)$$

where $C = C_1\sqrt{\alpha}(1-\sqrt{\alpha})^2$. Equations (5) and (6) satisfy Equation (2). S in Equations (5) and (6) can be a function of the steam quality and void fraction, and it is supposed that

$$0\leq S\leq 1 \qquad (7)$$

because F_{gt} is larger when there is vaporization than when there is no vaporization and F_{lt} is smaller under vaporization than under no vaporization.

In Equations (1) and (3) F_{gt} and F_{lt} are substituted by Equations (5) and (6), and d/dL is substituted by

$$\frac{d}{dL}=\frac{dP}{dL}\frac{\partial}{\partial P}+\frac{dQ}{dL}\frac{\partial}{\partial Q} \qquad (8)$$

Equations (1) and (3) become

$$\frac{1}{g}G_l\left(\frac{dP}{dL}\frac{\partial V_l}{\partial p}+\frac{dQ}{dL}\frac{\partial V_l}{\partial Q}\right)=A_l\frac{dP}{dL}-A_l\rho_l\sin\theta-\frac{1}{g}A\,F_{wl}+\frac{1}{g}C\,V_R^2-\frac{1}{g}(1-S)V_R\left(\frac{dP}{dL}\frac{\partial G_g}{\partial p}+\frac{dQ}{dL}\frac{\partial G_g}{\partial Q}\right) \qquad (9)$$

$$\frac{1}{g}G_g\left(\frac{dP}{dL}\frac{\partial V_g}{\partial p}+\frac{dQ}{dL}\frac{\partial V_g}{\partial Q}\right)=A_g\frac{dP}{dL}-A_g\rho_g\sin\theta-\frac{1}{g}C\,V_R^2-\frac{1}{g}S\,V_R\left(\frac{dP}{dL}\frac{\partial G_g}{\partial P}+\frac{dQ}{dL}\frac{\partial G_g}{\partial Q}\right) \qquad (10)$$

Equations (9) and (10) are rearranged into

$$\frac{1}{g}\frac{dP}{dL}\left[gA_l-G_l\frac{\partial V_l}{\partial p}-(1-S)V_R\frac{\partial G_g}{\partial p}\right]=A_l\rho_l\sin\theta+\frac{1}{g}AF_{wl}-\frac{1}{g}C\,V_R^2+\frac{1}{g}\frac{dQ}{dL}\left[G_l\frac{\partial V_l}{\partial Q}+(1-S)V_R\frac{\partial G_g}{\partial Q}\right] \qquad (11)$$

$$\frac{1}{g}\frac{dP}{dL}\left[gA_g-G_g\frac{\partial V_g}{\partial p}-S\,V_R\frac{\partial G_g}{\partial p}\right]=A_g\rho_g\sin\theta+\frac{1}{g}C\,V_R^2+\frac{1}{g}\frac{dQ}{dL}\left[G_g\frac{\partial V_g}{\partial Q}+S\,V_R\frac{\partial G_g}{\partial Q}\right] \qquad (12)$$

Eliminating dP/dL from Equations (11) and (12) one gets

$$AA_gF_{wl}-C\,V_R^2(A_g+A_l)+g\,A_lA_g(\rho_l-\rho_g)\sin\theta+\frac{dQ}{dL}\left[-A_lG_g\frac{\partial V_g}{\partial Q}+A_gG_l\frac{\partial V_l}{\partial Q}-(SA-A_g)V_R\frac{\partial G_g}{\partial Q}\right]=\frac{1}{g}(gA_l\rho_l\sin\theta+AF_{wl}-C\,V_R^2)\left[G_g\frac{\partial V_g}{\partial p}+SV_R\frac{\partial G_g}{\partial p}\right]-\frac{1}{g}(gA_g\rho_g\sin\theta+C\,V_R^2)\left[G_l\frac{\partial V_l}{\partial p}+(1-S)\,V_R\frac{\partial G_g}{\partial p}\right] \qquad (13)$$

THE FIRST APPROXIMATION FOR HORIZONTAL FLOW WITH NO HEAT ADDITION

When the two-phase flow is horizontal and with no heat addition, $\sin\theta = 0$ and $dQ/dL = 0$, and Equation (13) reduces to

$$AA_gF_{wl}-CAV_R^2=\frac{1}{g}(AF_{wl}-CV_R^2)\left(G_g\frac{\partial V_g}{\partial p}+SV_R\frac{\partial G_g}{\partial p}\right)-\frac{1}{g}CV_R^2\left[G_l\frac{\partial V_l}{\partial p}+(1-S)V_R\frac{\partial G_g}{\partial p}\right] \qquad (14)$$

Equation (14) is rewritten by substituting

$$A_g=A\,\alpha \qquad (15)$$

$$A_l=A\,(1-\alpha) \qquad (16)$$

$$G_g=Gx \qquad (17)$$

$$G_l=G(1-x) \qquad (18)$$

$$V_g=\frac{G}{A}\frac{1}{\rho_g}\frac{x}{\alpha}=\frac{G}{A}\frac{1}{\rho_l}m\frac{x}{\alpha} \qquad (19)$$

$$V_l=\frac{G}{A}\frac{1}{\rho_l}\left(\frac{1-x}{1-\alpha}\right) \qquad (20)$$

$$V_R=V_g-V_l=\frac{G}{A}\frac{1}{\rho_l}\left(m\frac{x}{\alpha}-\frac{1-x}{1-\alpha}\right) \qquad (21)$$

$$F_{wl}=\frac{\lambda}{2D}\frac{G^2}{A^2}\frac{1}{\rho_l}\left(\frac{1-x}{1-\alpha}\right)^2 \qquad (22)$$

$$\frac{\partial V_g}{\partial p}=\left(\frac{\partial V_g}{\partial x}+\frac{\partial V_g}{\partial \alpha}\frac{\partial \alpha}{\partial x}\right)\frac{\partial x}{\partial p}+\left(\frac{\partial V_g}{\partial m}+\frac{\partial V_g}{\partial \alpha}\frac{\partial \alpha}{\partial m}\right)\frac{dm}{dp}=\frac{G}{A\rho_l}\left[m\left(\frac{1}{\alpha}-\frac{x}{\alpha^2}\frac{\partial \alpha}{\partial x}\right)\frac{\partial x}{\partial p}+\left(\frac{x}{\alpha}-m\frac{x}{\alpha^2}\frac{\partial \alpha}{\partial m}\right)\frac{dm}{dp}\right] \qquad (23)$$

$$\frac{\partial V_l}{\partial p}=\frac{G}{A\rho_l}\left[\left(-\frac{1}{1-\alpha}+\frac{1-x}{(1-\alpha)^2}\frac{\partial \alpha}{\partial x}\right)\frac{\partial x}{\partial p}+\frac{1-x}{(1-\alpha)^2}\frac{\partial \alpha}{\partial m}\frac{dm}{dp}\right] \qquad (24)^*$$

* $(\partial\rho_l/\partial p)$ is neglected because it is very small compared with $(\partial\rho_g/\partial p)$.

$$\frac{\partial G_g}{\partial p} = G\frac{\partial x}{\partial p} \tag{25}$$

$$\frac{\lambda}{2D}\frac{\rho_l A}{C} = k \tag{26}$$

Equation (14) then becomes

$$k\alpha \quad \left(\frac{1-x}{1-\alpha}\right)^2 - \left(m\frac{x}{\alpha} - \frac{1-x}{1-\alpha}\right)^2 =$$

$$\frac{G^2}{gA^2\rho_l}\left[k\left(\frac{1-x}{1-\alpha}\right)^2 - \left(m\frac{x}{\alpha} - \frac{1-x}{1-\alpha}\right)^2\right]$$

$$\left\{\left[m\left(\frac{x}{\alpha} - \frac{x^2}{\alpha^2}\frac{\partial\alpha}{\partial x}\right) + S\left(m\frac{x}{\alpha} - \frac{1-x}{1-\alpha}\right)\right]\right.$$

$$\left.\frac{\partial x}{\partial p} + \left(\frac{x^2}{\alpha} - m\frac{x^2}{\alpha^2}\frac{\partial\alpha}{\partial m}\right)\frac{dm}{dp}\right\} -$$

$$\frac{G^2}{gA^2\rho_l}\left(m\frac{x}{\alpha} - \frac{1-x}{1-\alpha}\right)^2\left\{\left[-\left(\frac{1-x}{1-\alpha}\right) + \right.\right.$$

$$\left.\left(\frac{1-x}{1-\alpha}\right)^2\frac{\partial\alpha}{\partial x} + (1-S)\left(m\frac{x}{\alpha} - \frac{1-x}{1-\alpha}\right)\right]$$

$$\left.\frac{\partial x}{\partial p} + \left(\frac{1-x}{1-\alpha}\right)^2\frac{\partial\alpha}{\partial m}\frac{dm}{dp}\right\} \tag{27}$$

The right side of Equation (27) is much smaller than each term in the left side when inlet velocity $G/\rho_l A$ is less than 30 ft./sec. and thus may be neglected for the first approximation. In this manner Equation (27) reduces to

$$k\alpha\left(\frac{1-x}{1-\alpha}\right)^2 - \left(m\frac{x}{\alpha} - \frac{1-x}{1-\alpha}\right)^2 = 0 \tag{28}$$

Equation (28) is rewritten to give

$$x = \frac{\alpha(1 \pm \sqrt{k}\sqrt{\alpha})}{m(1-\alpha) + \alpha \pm \sqrt{k}\,\alpha\sqrt{\alpha}} \tag{29}$$

where

$$k = \frac{\lambda\rho_l A}{2D\,C_1}\frac{1}{\sqrt{\alpha}\,(1-\sqrt{\alpha})^2} = \frac{k_1}{\sqrt{\alpha}\,(1-\sqrt{\alpha})^2} \tag{30}$$

The plus sign is taken for the sign before the square root; otherwise x becomes negative for some values of α.

Since C_1 in Equation (30) is considered to be a function of turbulence at the interface between the phases, it depends on surface tension, viscosity, density, and also channel geometry. λ in the equation is also considered to be a function of void fraction and turbulence because it depends on the velocity distribution in the liquid phase. The functional form of k_1 may be determined by more precise experiments for interfacial conditions and turbulences.

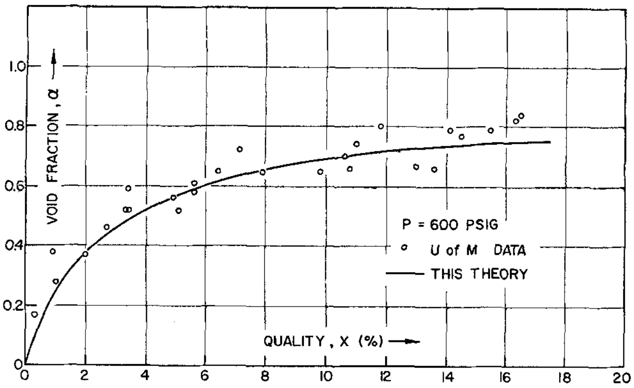

Fig. 3. Comparison of the theory with University of Minnesota data for horizontal flow without heat addition.

For simplicity in this paper the dependence of k_1 on the channel geometry is neglected, and the functional form of k_1 is assumed as follows:

$$k_1 = \frac{\lambda}{2D}\frac{\rho_l A}{C_1} = \frac{a}{P} \tag{31}$$

a is a constant determined from the results of experiments at the University of Minnesota (*4, 5*):

$$a = 10 \tag{32}$$

where P is measured in pounds per square inch absolute.

When $m(1-\alpha)$ is much larger than α, Equation (29) is approximately written as

$$x = \frac{\alpha}{m(1-\alpha)}\left[1 + \frac{\alpha^{1/4}}{1-\sqrt{\alpha}}\sqrt{k_1}\right] \tag{33}$$

The U of M test results and the calculated values from Equation (33) are shown in Figure 2 for 1,000 lb./sq.in. gauge and Figure 3 for 600 lb./sq. in. gauge.

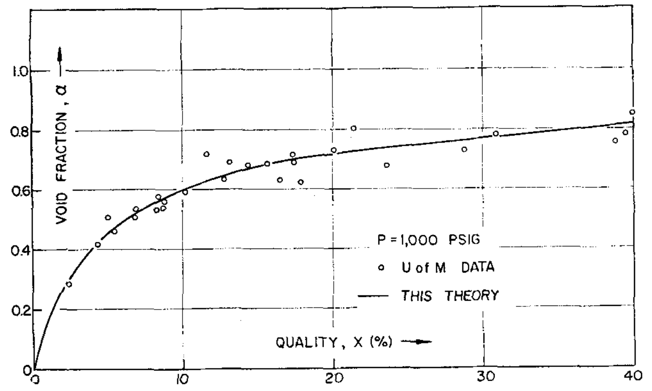

Fig. 2. Comparison of the theory with University of Minnesota data for horizontal flow without heat addition.

THE SECOND APPROXIMATION FOR HORIZONTAL FLOW WITH NO HEAT ADDITION

In the second approximation for horizontal flow with no heat addition the following equation is assumed:

$$x = F(\alpha) + \epsilon_p(\alpha) \tag{34}$$

where $F(\alpha)$ is given by Equation (29), and ϵ_p is so small that the square of $\epsilon_p(\alpha)$ and the products of $\epsilon_p(\alpha)$ with $(G^2/gA^2\rho_l)(\partial x/\partial P)$ and $(G^2/gA^2\rho_l)(dm/dp)$ may be neglected as well as the square and the products of $(G^2/gA^2\rho_l)(\partial x/\partial P)$ and $(G^2/gA^2\rho_l)(dm/dp)$.

Substituting Equation (34) into Equation (27) and neglecting small quantities one can obtain the following equation for $\epsilon_p(\alpha)$:

$$\epsilon_p(\alpha) = -\frac{G^2}{g\rho_l A}\left\{\frac{\partial x}{\partial p}\frac{\sqrt{k\alpha}}{2(1-\alpha)}\right.$$

$$\left[\frac{\alpha}{1 + \frac{3}{2}\sqrt{k\alpha} - \frac{1}{2}\sqrt{k\alpha}\,\alpha} - \right.$$

$$\left.\frac{1 + \sqrt{k\alpha} - 2\sqrt{k\alpha}\,\alpha + S\sqrt{k\alpha}}{m(1-\alpha)}\right] +$$

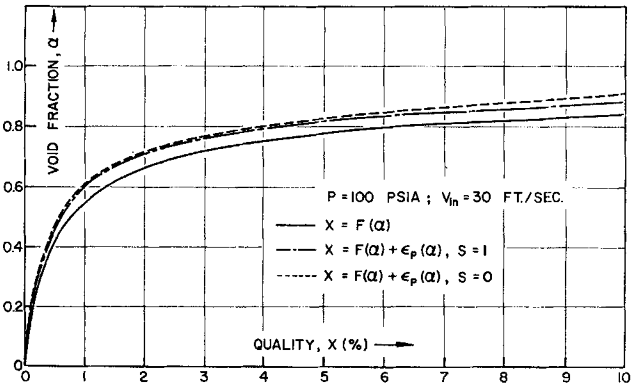

Fig. 4. Second approximation for horizontal flow without heat addition.

$$\frac{1}{\rho_l}\frac{\partial \rho_g}{\partial p}\frac{\sqrt{k\alpha}\ \alpha(1+\sqrt{k\alpha})}{2(1-\alpha)^2\left(1+\frac{3}{2}\sqrt{k\alpha}-\frac{1}{2}\sqrt{k\alpha\ \alpha}\right)}$$
$$\left[\alpha-\frac{1}{m}(1+\sqrt{k\alpha})\left(\alpha+\frac{1}{2}\sqrt{k\alpha}+\frac{1}{2}\sqrt{k\alpha\ \alpha}\right)\right]\Bigg\} \quad (35)^*$$

Equation (34) is calculated and shown in Figure 4 for the case where P = 100 lb./sq.in.abs., V_{in} = 30 ft./sec. S in Equation (35) may be a function of steam quality and void fraction, as stated before. But it does not affect significantly the values of ϵ_p as can be seen from Figure 4 where both cases for $S = 1$ and $S = 0$ are shown.

Since k_1 increases as the system pressure decreases, the effects of the momentum change accompanied by pressure drop on the relation between steam quality and void fraction become more significant as the system pressure decreases.

APPROXIMATION FOR HORIZONTAL FLOW WITH HEAT ADDITION

The correction term $\epsilon_Q(\alpha)$ for heat addition is obtained from the following equation which is obtained from Equation (13) by making $\sin\theta = 0$ and neglecting the right side of the equation:

$$\alpha A^2F_{wl} - ACV_R^2 = \frac{dQ}{dL}\left[A_lG_g\frac{\partial V_g}{\partial Q} - A_gG_g\frac{\partial V_l}{\partial Q} + (SA-A_g)V_R\frac{\partial G_g}{\partial Q}\right] \quad (36)$$

Equation (36) is rewritten by substituting Equations (15) to (22) and the following:

$$\frac{\partial V_g}{\partial Q} = \frac{G}{A\rho_l}m\left(\frac{1}{\alpha}-\frac{x}{\alpha^2}\frac{\partial\alpha}{\partial x}\right)\frac{\partial x}{\partial Q} \quad (37)$$

$$\frac{\partial V_l}{\partial Q} = \frac{G}{A\rho_l}\left(\frac{-1}{1-\alpha}+\frac{1-x}{(1-\alpha)^2}\frac{\partial\alpha}{\partial x}\right)\frac{\partial x}{\partial Q} \quad (38)$$

$$\frac{\partial G_g}{\partial Q} = G\frac{\partial x}{\partial Q} \quad (39)$$

Equation (36) becomes†

$$k\alpha\left(\frac{1-x}{1-\alpha}\right)^2 - \left(m\frac{x}{\alpha}-\frac{1-x}{1-\alpha}\right)^2 = \frac{\rho_lA}{C}\cdot\frac{\partial x}{\partial Q}\cdot\frac{dQ}{dL}\left[m(1-\alpha)\left(\frac{x}{\alpha}-\frac{x^2}{\alpha^2}\frac{\partial\alpha}{\partial x}\right) + \alpha\left\{\frac{1-x}{1-\alpha}-\left(\frac{1-x}{1-\alpha}\right)^2\frac{\partial\alpha}{\partial x}\right\} + (S-\alpha)\left(m\frac{x}{\alpha}-\frac{1-x}{1-\alpha}\right)\right] \quad (40)$$

Assuming ϵ_Q as well as $(\rho_lA/C)(dx/dL)$ is so small that the square and their products may be neglected, and also assuming

$$x = F(\alpha) + \epsilon_Q(\alpha) \quad (41)$$

one gets an equation for $\epsilon_Q(\alpha)$:

$$\epsilon_Q(\alpha) = \frac{\rho_lA}{C}\frac{\partial x}{\partial Q}\frac{dQ}{dL}\left[\frac{\sqrt{k}}{2}\frac{\alpha\sqrt{\alpha}(1-\alpha)}{1+\frac{3}{2}\sqrt{k\alpha}-\frac{1}{2}\sqrt{k\alpha}\ (\alpha)} - \frac{\alpha\sqrt{\alpha}+S\sqrt{k\alpha}}{2m\sqrt{k}}\right] \quad (42)$$

The value of k for the flow with heat addition may be different from the value for no heat addition because k depends on the turbulence and the turbulence can be different from that for no heat addition. To determine the value of k for the flow with heat addition more precise experiments may be needed.

Since there are few available data to determine the value of k for horizontal flow with heat addition, the same value as that for the flow with no heat addition is used in the present calculations. In Figure 5 calculated values of Equation (41) are shown for the case where the system pressure is 100 lb./sq.in.abs., the heat input is 100 kw./liter of fluid, and the inlet velocity is ½ ft./sec.

When the effect of right side of Equation (13) is not negligible, the correction term ϵ_p stated in the former section should be added to Equation (41).

THE FIRST APPROXIMATION FOR VERTICAL FLOW

When the two-phase flow is vertical with no heat addition, and the inlet velocity is so small that the right side of Equation (13) is negligible, Equation (13) reduces to

$$AA_gF_{wl} - ACV_R^2 + gA_lA_g(\rho_l-\rho_g) = 0 \quad (43)$$

Substituting Equations (15) to (22) into Equation (43) one gets

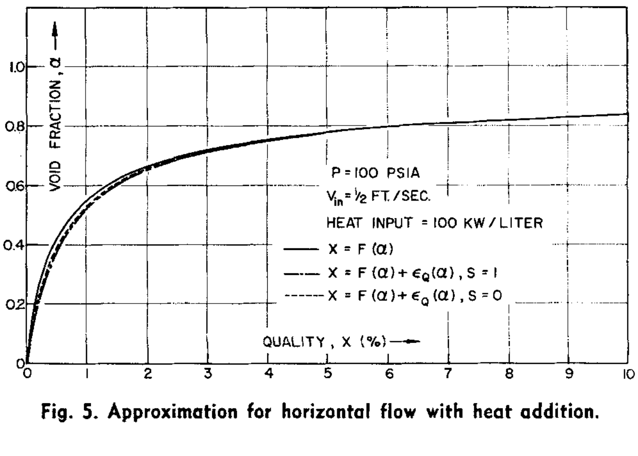

Fig. 5. Approximation for horizontal flow with heat addition.

* See Appendix as for calculations of $(\partial x/\partial p)$ and $(\partial\rho_g/\partial p)$.

† Levy (*1*) derived an equation similar to Equation (40) and a relation between the steam quality and void fraction by assuming that the sum of the terms in the bracket of the right side of Equation (40) is zero. The left side is close to zero when the heat input is small, because (ρ_lA/C) (dx/dL) is very small and not because the sum of the terms in the bracket is small.

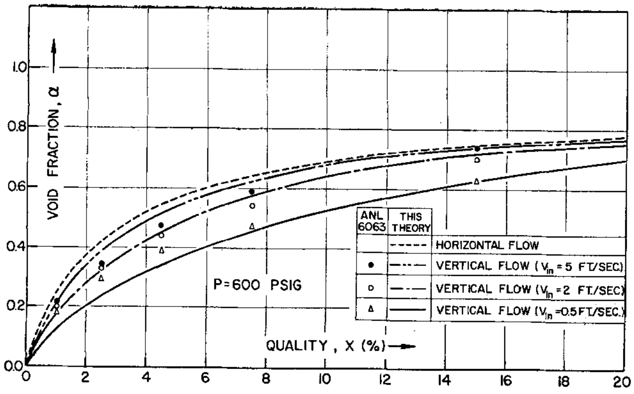

Fig. 6. The effect of the mass flow rate and the difference between vertical and horizontal flow (with heat addition).

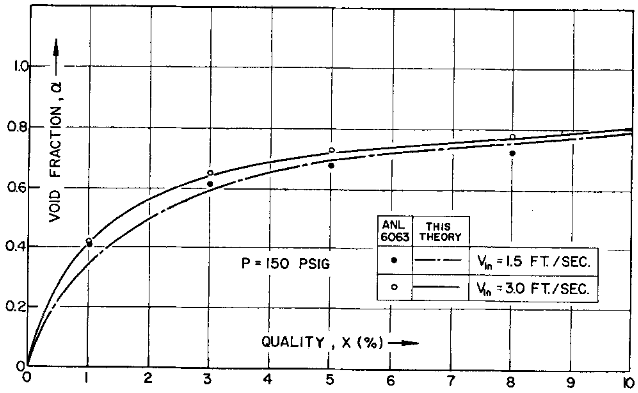

Fig. 7. The effect of the mass flow rate on vertical flow (with heat addition).

$$k\alpha\left(\frac{1-x}{1-\alpha}\right)^2-\left(m\frac{x}{\alpha}-\frac{1-x}{1-\alpha}\right)^2+N\alpha(1-\alpha)=0 \tag{44}$$

where

$$N=\frac{gA^3\rho_l^2\,(\rho_l-\rho_g)}{C_1G^2\sqrt{\alpha}\,(1-\sqrt{\alpha})^2}=\frac{N_1}{\sqrt{\alpha}\,(1-\sqrt{\alpha})^2} \tag{45}$$

Equation (44) is rewritten to yield

$$x=\frac{\alpha(m-m\alpha+\alpha)-k\alpha^3+\sqrt{km^2\alpha^3\,(1-\alpha)^2+N\alpha^3\,(1-\alpha)^3\,(m(1-\alpha)+\alpha)^2-Nk\alpha^6\,(1-\alpha)^3}}{(m-m\alpha+\alpha)^2-k\alpha^3} \tag{46}$$

When $m(1-\alpha)$ is much larger than α, Equation (46) is approximately

$$x=\frac{\alpha}{m(1-\alpha)}\left[1+\frac{\alpha^{1/4}}{1-\sqrt{\alpha}}\sqrt{k_1+N_1(1-\alpha)^3}\right] \tag{47}$$

N_1 is calculated from Equations (31) and (45):

$$N_1=\left(1-\frac{1}{m}\right)\frac{2gD}{\lambda\dfrac{G^2}{\rho_l^2A^2}}\,(k_1)$$

Thus Equation (47) becomes

$$x=\frac{\alpha}{m(1-\alpha)}\left[1+\frac{\alpha^{1/4}}{1-\sqrt{\alpha}}\sqrt{k_1}\sqrt{1+\frac{2gD}{\lambda\,V_{in}^2}\,(1-\alpha)^3}\right] \tag{48}$$

The turbulence in vertical flow can be different from that in horizontal flow, and therefore the value of k_1 for vertical flow can be different from that for horizontal flow; however for the calculations in this paper the same value of k_1 is used.

As can be seen from Equation (46) or (48) the relation between the steam quality and the void fraction for vertical flow depends on the mass flow rate and differs from the relation for horizontal flow. In Figures 6 and 7 the differences of void fraction between horizontal and vertical flow and the effects of the mass flow rate on void fraction calculated from this theory as well as from the *ANL* working curves (*6*) are shown. In Figure 8 calculated values from this theory and the experimental values by Cook (*7*) are shown.

For the case where the effect of flashing and heat addition is not negligible, similar correction terms as ϵ_P and ϵ_Q which are stated in the former sections for horizontal flow are added to Equation (46) or (48).

FRICTIONAL LOSS IN TWO-PHASE FLOW

The frictional loss in two-phase flow which is expressed as $(dP/dL)_{TP}$ by Martinelli (*8*) is calculated from Equation (22). $(dP/dL)_L$ in Martinelli's theory is given by

$$\left(\frac{dP}{dL}\right)_L=\frac{\lambda}{2D}\frac{G^2}{A^2}\frac{1}{\rho_l}\,(1-x)^2 \tag{49}$$

Therefore the ratio $(dP/dL)_{TP}/(dP/dL)_L$ is written as

$$\frac{\left(\dfrac{dP}{dL}\right)_{TP}}{\left(\dfrac{dP}{dL}\right)_L}=\frac{1}{(1-\alpha)^2} \tag{50}$$

In Figure 9 the calculated value from Equation (50) and test data obtained at the University of Minnesota are shown.

The reason why the difference between the calculated and the experimental value increases when the steam quality increases may be explained as follows. In Equations (22) and (49) the same value for λ is used, in spite of the fact that λ for two-phase flow may be a function of void fraction and other flow conditions. If the velocity distribution in the liquid film is measured, and the functional form of λ is determined, the agreement may be improved.

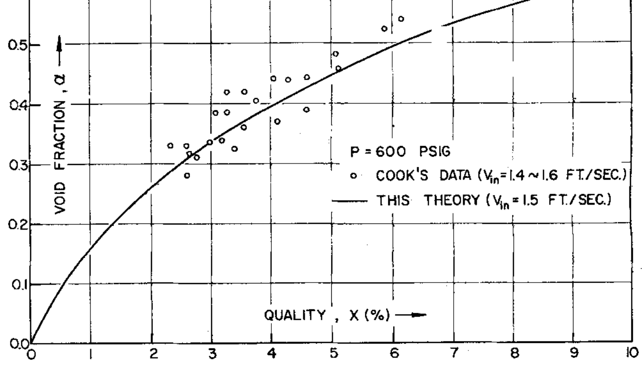

Fig. 8. Comparison of the theory with Cook's data for vertical flow with heat addition).

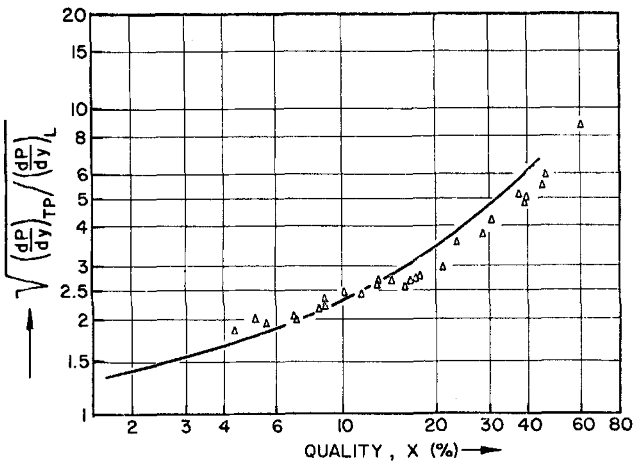

Fig. 9. Comparison of the theory with University of Minnesota data at 1,000 lb./sq. in. gauge without heat addition.

SUMMARY

A three-region annular flow model has been proposed to predict the relationships between void fraction and quality for steam-water flows. The nature of the forces between regions for evaporating two-phase flow systems has been assumed. Relations have been developed for horizontal and vertical flows with and without heat addition. One empirical constant has been used, and the agreement between predicted and experimental results may be further improved by the introduction of refinements of the functional forms for the defining relations for the interfacial forces and friction factor. The effect of evaporation on the relation between steam quality and void fraction is not predominant if the inlet velocity does not exceed 30 ft./sec. or heat addition does not exceed 100 kw./liter.

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NOTATION

A = cross-sectional area of channel
A_g = cross-sectional area occupied by gas phase
A_l = cross-sectional area occupied by liquid phase
a = constant in Equation (31)
C = $C_1\sqrt{\alpha}(1-\sqrt{\alpha})^2$
C_1 = parameter in Equation (4)
D = pipe diameter of equivalent hydraulic diameter
F_{wl} = force between wall and liquid phase per cross-sectional area per unit length of pipe
F_{gt} = force between region B and C per unit length of pipe
F_{lt} = force between region A and B per unit length of pipe
G = total mass flow rate
G_g = mass flow rate of gas phase
G_l = mass flow rate of liquid phase
g = gravitational constant
h_g = specific enthalpy of gas phase
h_l = specific enthalpy of liquid phase
k = $k_1/\sqrt{\alpha}(1-\sqrt{\alpha})^2$
k_1 = dimensionless parameter = $\lambda\,\rho_l A/2D\,C_1$
L = length along the channel
m = ρ_l/ρ_g
N = $N_1/\sqrt{\alpha}(1-\sqrt{\alpha})^2$
N_1 = dimensionless parameter = $gA^3\rho_l^2(\rho_l-\rho_g)/G^2C_1$
P = static pressure
Q = heat generation per unit length of pipe
V_g = mean velocity of gas phase
V_l = mean velocity of liquid phase
V_R = $V_g - V_l$
V_{in} = $G/\rho_l A$ = inlet velocity
x = G_g/G = steam quality

Greek Letters

α = void fraction
λ = friction factor
ρ_g = density of gas phase
ρ_l = density of liquid phase
θ = angle of inclination of channel

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APPENDIX

For adiabatic two-phase flow system the following equation holds:

$$\Delta(G_l h_l) + \Delta(G_g h_g) = 0 \qquad (A.1)$$

Since $\Delta G_g = -\Delta G_l$, Equation (A.1) reduces to

$$(h_g - h_l)\,\Delta G_g = -(G_l \Delta h_l + G_g \Delta h_g) \qquad (A.2)$$

Substituting $G_l = G(1-x)$, $G_g = Gx$, one obtains the following equation for $(\partial x/\partial P)$:

$$\frac{\partial x}{\partial P} = \frac{1}{G}\frac{\partial G_g}{\partial P} = -\frac{1}{h_g - h_l}\left[(1-x)\frac{dh_l}{dp} + x\frac{dh_g}{dp}\right] \qquad (A.3)$$

If the pressure of a saturated vapor-water system decreases adiabatically, the difference of the gas density $-\Delta\rho_g$ is given as

$$-\Delta\rho_g = -\left(\frac{\partial\rho_g}{\partial P}\right)_{SAT}\Delta P - \left(\frac{\partial\rho_g}{\partial T}\right)_P \Delta T \qquad (A.4)$$

where $(\partial\rho_g/\partial P)_{SAT}\cdot\Delta P$ means the difference of gas density which is obtained when the system is changed in the saturated condition, and ΔT is given as

$$\Delta T = \frac{\Delta h}{C_p} \qquad (A.5)$$

Δh is the difference of the specific enthalpy of the saturated vapor corresponding to the pressure difference, and C_p is the constant pressure specific heat of the gas.

From Equations (A.4) and (A.5) the following equation for $(\partial\rho_g/\partial p)$ for adiabatic pressure decrease is obtained:

$$\frac{\partial\rho_g}{\partial p} = \left(\frac{\partial\rho_g}{\partial P}\right)_{SAT} + \frac{1}{C_p}\left(\frac{\partial\rho_g}{\partial T}\right)_P\left(\frac{\partial h}{\partial P}\right)_{SAT} \qquad (A.6)$$